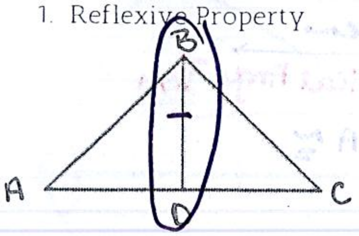
SAS SSS HL Hon
AAS . ASA

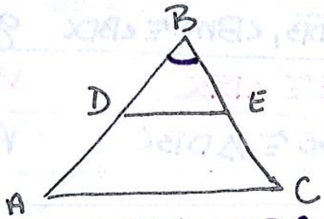
Geometry Proofs – Congruency in Triangles Notes

The 4 most common concepts to HELP prove triangles congruent:

1. Reflexive Property

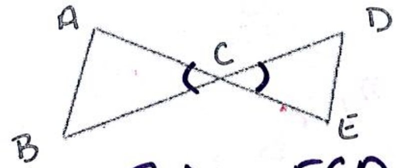


$\overline{BD} \cong \overline{BD}$



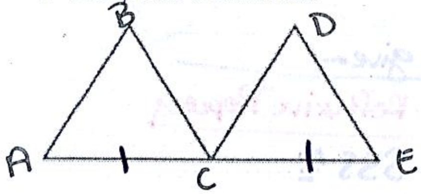
$\angle DBE \cong \angle ABC$

2. Vertical Angles Theorem



$\angle ACB \cong \angle ECD$

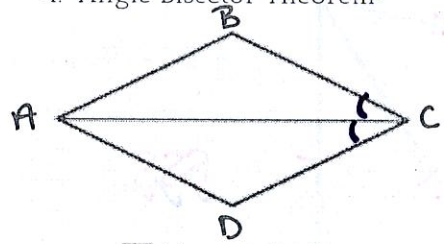
3. Midpoint Definition



Point C is the midpoint of $\overline{AE}$

$\overline{AC} \cong \overline{EC}$

4. Angle Bisector ~~Theorem~~ Definition

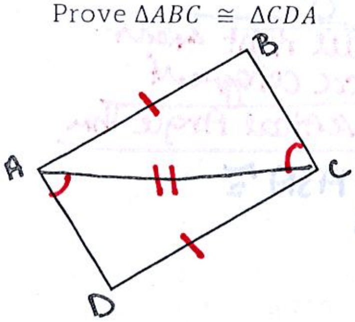


$\overline{AC}$ bisects $\angle BCD$

$\angle BCA \cong \angle DCA$

State if the two triangles are congruent. If congruent, write a two-column proof to prove it.

1. Given $\overline{AB} \cong \overline{DC}$, and $\angle CAD \cong \angle BCA$
   Prove $\Delta ABC \cong \Delta CDA$



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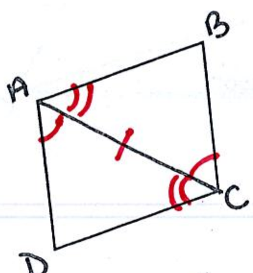
inconclusive

Statement	Reason

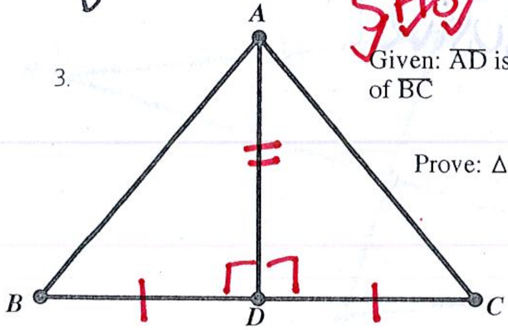
2. Given $\angle CAD \cong \angle BCA$ and $\angle ACD \cong \angle CAB$
   Prove $\Delta ABC \cong \Delta CDA$



ASA ✓

Statement	Reason
$\angle CAD \cong \angle BCA$, $\angle ACD \cong \angle CAB$	given
$\overline{AC} \cong \overline{AC}$	Reflexive Property
$\Delta ABC \cong \Delta CDA$	ASA $\cong$

3.



SAS ✓

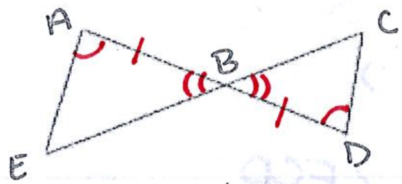
Given: $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$

Prove: $\Delta ABD \cong \Delta ACD$

Statement	Reason
$\overline{AD}$ is perpendicular bisector of $\overline{BC}$	given
$\angle BDA$ & $\angle CDA$ are right angles, $\overline{BD} \cong \overline{CD}$	definition of perpendicular bisector
$\angle BDA \cong \angle CDA$	all right angles are congruent
$\overline{AD} \cong \overline{AD}$	Reflexive Property
$\Delta ABD \cong \Delta ACD$	SAS $\cong$

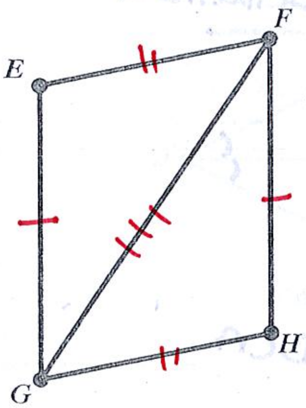
4. Given $\overline{AB} \cong \overline{DB}$, and $\angle BAE \cong \angle BDC$
Prove $\Delta ABE \cong \Delta DBC$

ASA



Statement	Reason
$\overline{AB} \cong \overline{DB}$, $\angle BAE \cong \angle BDC$	given
$\angle ABE \cong \angle DBC$	Vertical Angle Thm
$\Delta ABE \cong \Delta DBC$	ASA $\cong$

5.



Given: EG ≅ HF, and EF ≅ HG

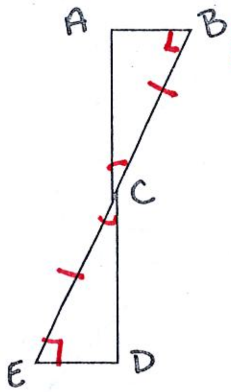
Prove: ΔEFG ≅ ΔHGF

SSS

Statement	Reason
$\overline{EG} \cong \overline{HF}$, $\overline{EF} \cong \overline{HG}$	given
$\overline{FG} \cong \overline{FG}$	Reflexive Property
$\Delta EFG \cong \Delta HGF$	SSS $\cong$

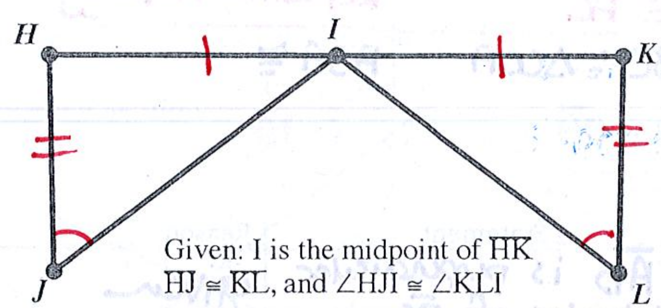
6. Given $\overline{BC} \cong \overline{EC}$, and $\angle B$ *and* $\angle E$ are right angles.
Prove $\Delta BCA \cong \Delta ECD$

ASA



Statement	Reason
$\overline{BC} \cong \overline{EC}$, $\angle B$ and $\angle E$ are right angles	given
$\angle B \cong \angle E$	all right angles are congruent
$\angle ACB \cong \angle DCE$	Vertical Angle Thm
$\Delta BCA \cong \Delta ECD$	ASA $\cong$

7.

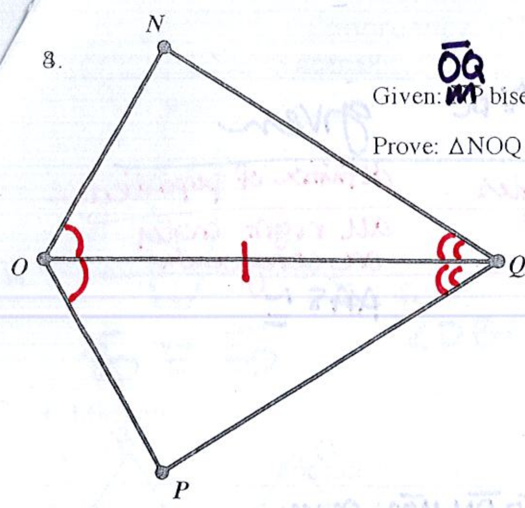


Given: I is the midpoint of HK
HJ ≅ KL, and ∠HJI ≅ ∠KLI

Prove: ΔHIJ ≅ ΔKIL

Statement	Reason
inconclusive	

8.



Given: $\overline{OQ}$ bisects ∠NOP and ∠NQP

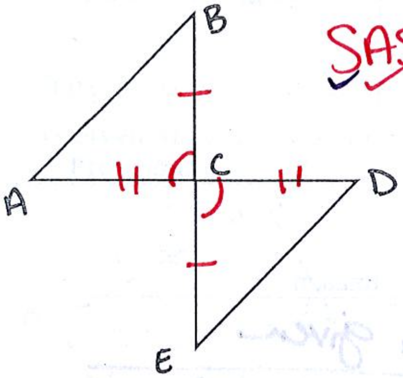
Prove: ΔNOQ ≅ ΔPOQ

ASA

Statement	Reason
$\overline{OQ}$ bisects ∠NOP ; ∠NQP	given
∠NOQ ≅ ∠POQ ∠NQO ≅ PQO	definition of angle bisector
$\overline{OQ} \cong \overline{OQ}$	Reflexive Property
ΔNOQ ≅ ΔPOQ	ASA ≅

9. Given $\overline{BC} \cong \overline{EC}$ and $\overline{AC} \cong \overline{DC}$
Prove $\Delta ACB \cong \Delta DCE$

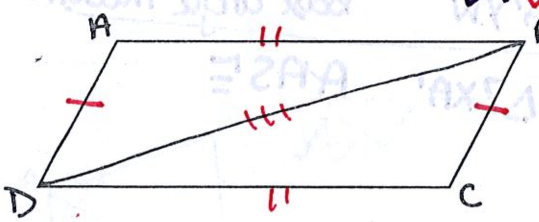
SAS



Statement	Reason
$\overline{BC} \cong \overline{EC}$, $\overline{AC} \cong \overline{DC}$	given
∠ACB ≅ ∠DCE	Vertical Angle Thm
ΔACB ≅ ΔDCE	SAS ≅

10. Given $\overline{AD} \cong \overline{CB}$ and $\overline{AB} \cong \overline{CD}$
Prove $\Delta DAB \cong \Delta BCD$

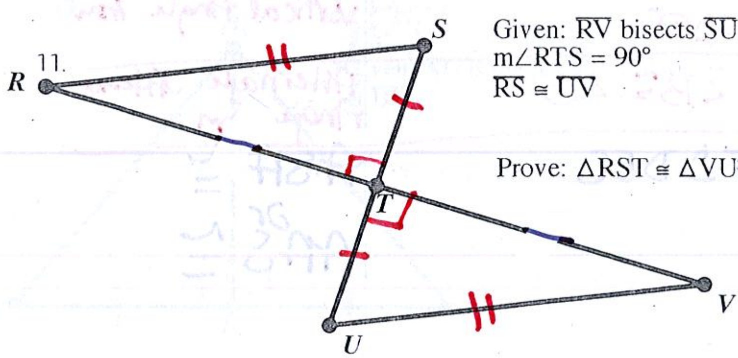
SSS



Statement	Reason
$\overline{AD} \cong \overline{CB}$, $\overline{AB} \cong \overline{CD}$	given
$\overline{DB} \cong \overline{DB}$	Reflexive Property
ΔDAB ≅ ΔBCD	SSS ≅

HL

11.



Given: $\overline{RV}$ bisects $\overline{SU}$
m∠RTS = 90°
$\overline{RS} \cong \overline{UV}$

Prove: ΔRST ≅ ΔVUT

Statement	Reason
$\overline{RV}$ bisects $\overline{SU}$, m∠RTS = 90 $\overline{RS} \cong \overline{UV}$	given
$\overline{ST} \cong \overline{UT}$	definition of bisector
m∠UTV = 90°	Vertical Angle Theorem
ΔRST ≅ ΔVUT	HL ≅

12. Given $\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$, $\overline{AC} \cong \overline{DC}$ and $\angle BAC \cong \angle EDC$

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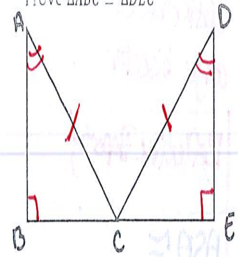
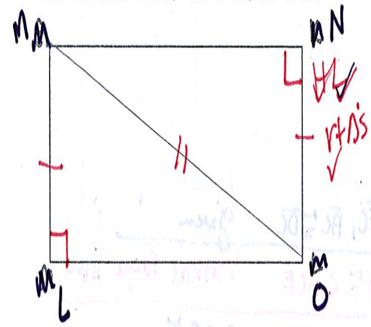
Prove $\Delta ABC \cong \Delta DEC$



Statement	Reason
$\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$, $\overline{AC} \cong \overline{DC}$, $\angle BAC \cong \angle EDC$	given
$\angle B$ and $\angle E$ are right angles	definition of perpendicular
$\angle B \cong \angle E$	all right angles are congruent
$\Delta ABC \cong \Delta DEC$	AAS $\cong$

13. Given $\overline{LM} \cong \overline{NO}$, $\overline{ML} \perp \overline{LO}$, $\overline{ON} \perp \overline{NM}$



Statement	Reason
$\overline{LM} \cong \overline{NO}$, $\overline{ML} \perp \overline{LO}$, $\overline{ON} \perp \overline{NM}$	given
$\angle L$ and $\angle N$ are right angles	definition of perpendicular
$\overline{MO} \cong \overline{MO}$	Reflexive Property
$\Delta MLO \cong \Delta ONM$	HL $\cong$

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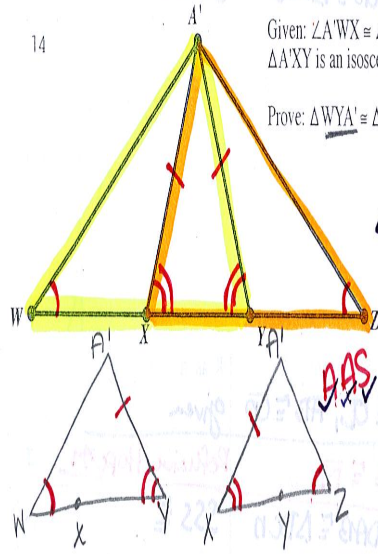
Given: $\angle A'WX \cong \angle A'ZY$

$\Delta A'XY$ is an isosceles triangle

Prove: $\Delta WYA' \cong \Delta ZXA'$

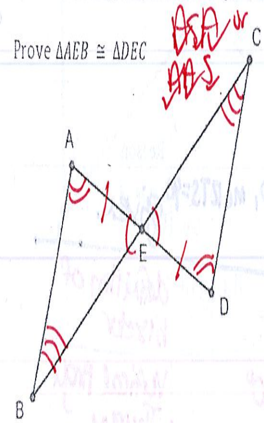


Statement	Reason
$\angle A'WX \cong \angle A'ZY$, $\Delta A'X'Y$ is isosceles triangle	given
$\overline{A'X} \cong \overline{A'Y}$	definition of isosceles triangle
$\angle A'XZ \cong \angle A'YW$	base angle theorem
$\Delta WYA' \cong \Delta ZXA'$	AAS $\cong$

15. Given: $\overline{AB} \parallel \overline{CD}$, E is the midpoint of $\overline{AD}$

Prove $\Delta AEB \cong \Delta DEC$

ASA or AAS



Statement	Reason
$\overline{AB} \parallel \overline{CD}$, E is midpoint of $\overline{AD}$	given
$\angle AEB \cong \angle DEC$	vertical Angle Thm
$\angle A \cong \angle D$, $\angle B \cong \angle C$	Alternate Interior Angle Thm
$\Delta AEB \cong \Delta DEC$	ASA $\cong$ or AAS $\cong$